THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050A Tutorial 4

1. Let $x_n := 1/1^2 + 1/2^2 + \cdots + 1/n^2$ for each $n \in \mathbb{N}$. Show that (x_n) is convergent.

2. (a) State the definition of Cauchy sequence.

   (b) Let $x_n := \sqrt{n}$ for $n \in \mathbb{N}$. Show directly from the definition that (x_n) is not a Cauchy sequence.

3. Let (x_n) be a contraction sequence of real numbers, that is, there exists a constant $C \in (0, 1)$ such that
$$|x_{n+2} - x_{n+1}| \leq C|x_{n+1} - x_n|$$
for all $n \in \mathbb{N}$.

   (a) Show that (x_n) is Cauchy and hence convergent.

   (b) Let $x_1 := 2$ and $x_{n+1} := 2 + 1/x_n$ for $n \geq 1$. Show that (x_n) is contractive. Find the limit of (x_n).

4. (a) Let (x_n) be a sequence of real numbers. State the definition of properly divergent sequence : $\lim(x_n) = +\infty$ or $\lim(x_n) = -\infty$.

   (b) Show that the sequence $(\sqrt{n^2 + 2})$ is properly divergent .

   (c) Show that if $\lim(a_n/n) = L$, where $L > 0$, then $\lim(a_n) = +\infty$.